Program name: fpd

Usage: fpd -o "output prefix" "input unit vector fields ..."

Function: For a given set of input *unit vector fields*, this program computes the *first principal direction* at each voxel.

We define a *unit vector field* to be vector-valued image, where the image value at each voxel is represented by a unit vector. The program expects the input unit vector fields to be in 4D ANALYZE format, with the fourth dimension equal to 3, representing the x, y, and z components of the vector field. The data type is expected to be *float*. All input vector fields are expected to have the same dimensions (i.e., the same number of image rows, columns, and slices). The data storage in the $*$.img file of the ANALYZE format is expected to be in such a way that the 4th dimension varies most slowly.

Suppose that you have specified n unit vector fields as inputs to the program. Thus, at a given voxel v, we have n unit vectors $\mathbf{I}_i$, $i = 1, \ldots, n$. We define the *first principal direction* at voxel v as the direction $\mathbf{a}$ ($\mathbf{a}^T\mathbf{a} = 1$) which minimizes the angles θ_i between $\mathbf{a}$ and $\mathbf{I}_i$, $i = 1, \ldots, n$; that is, if

$$Q = \sum_{i=1}^{n} \cos^2 \theta_i \tag{1}$$

is maximum. Let us define the (3×3) matrix $\mathbf{T}$ as follows:

$$\mathbf{T} = \sum_{i=1}^{n} \mathbf{I}_i \mathbf{I}_i^T. \tag{2}$$

Suppose that $\lambda_1, \lambda_2, \lambda_3$ are the three eigenvalues of $\mathbf{T}$ ($\lambda_1 > \lambda_2 > \lambda_3$). It can be shown that the first principal direction $\mathbf{a}$ of the set $\mathbf{I}_i$ is given by the normalized eigenvector corresponding to λ_1, the largest eigenvalue of $\mathbf{T}$. Furthermore, the sum of the three eigenvalues of $\mathbf{T}$ equals n, that is: $\lambda_1 + \lambda_2 + \lambda_3 = n$.

Program outputs: The program outputs the computed first principal direction map in 4D ANALYZE format. The output files for this image will be *prefix*.hdr and *prefix*.img. Further more, the program outputs the file *prefix*.vec in which the first principal direction map is stored such that the x, y, and z components vary faster than the voxel index. Basically, the *prefix*.img and *prefix*.vec files contain the same image information. The difference is that in *prefix*.img, the voxel index varies faster than the x, y, and z components, while in *prefix*.vec the opposite is true. The program also outputs a 3D ANALYZE image *prefix*L1.hdr and *prefix*L1.img with voxel values $100\lambda_1/(\lambda_1 + \lambda_2 + \lambda_3)\%$ or equivalently $(100\lambda_1/n)\%$. Finally the program outputs a binary mask in 3D ANALYZE format *prefix*_msk.hdr and *prefix*_msk.img indicating the voxels for which the first principal direction map was computed.

Reference: K.V. Mardia et al., "Multivariate Analysis", 1979, Academic Press.

Author: Babak A. Ardekani, PhD